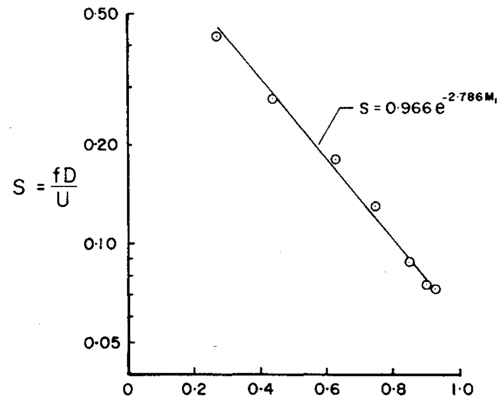

Fig. 5 Strouhal number vs Mach number.

boundary layer, may account for the difference in fluctuation magnitude for the two cases.

Although the equipment necessary to obtain detailed frequency information concerning the base pressure fluctuation was not available, some results can be drawn from the data. A simple count of the number of fluctuations in base pressure that occurred during an extended period of time yielded an average value for the frequency of the fluctuations. This average value of frequency gives a rough estimate of the frequencies at which that portion of the signal with significant amplitudes occur. In general, the average frequency decreased with increasing Mach number from approximately 2000 Hz at the low Mach numbers to approximately 1000 Hz at near-sonic speeds. A Strouhal number based on the average frequency, the model diameter, and the freestream velocity was computed. The Strouhal number as a function of Mach number is shown in Fig. 5. A linear relationship is evident on the semilog plot and a straight line, which was fit to the data by a least-squares method, is shown in the figure.

Conclusions

The base pressure fluctuations on an axisymmetric blunt-based body at subsonic speeds have been investigated. The tests were conducted over the entire subsonic Mach number range in a special wind tunnel that was free of support interference. The magnitudes of the fluctuations were found to be significant and increased with increasing Mach number. Magnitudes between 120 dB and 150 dB were observed. An average frequency and Strouhal number were found to decrease with increasing Mach number. Average frequencies between 1000 Hz and 2000 Hz were observed, while Strouhal numbers were between 0.05 and 0.5.

Acknowledgement

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References

[1]Merz, R.A., "The Turbulent Near-Wake of an Axisymmetric Blunt Based Body at Subsonic Speeds," Ph.D. Thesis, Mechanical, Industrial and Aerospace Engineering Department, Rutgers Univ., May 1975.

[2]McErlean, D.P., "The Near-Wake of a Blunt Based Axisymmetric Body at Mach .14," Ph.D. Thesis, Mechanical, Industrial and Aerospace Engineering Department, Rutgers Univ., Jan. 1970.

[3]Eldred, K.M., "Base Pressure Fluctuations," *The Journal of the Acoustical Society of America,* Vol. 33, Jan. 1961, pp. 59-63.

[4]Mabey, D.G., "Some Measurements of Base Pressure Fluctuations at Subsonic and Supersonic Speeds," Aeronautical Research Council, C.P. No. 1204, Aug. 1972.

[5]Merz, R.A., Page, R.H., and Przirembel, C.E.G., "Subsonic Axisymmetric Near-Wake Studies," *AIAA Journal,* Vol. 16, July 1978, pp. 656-662.

Some Exact Solutions to Guderley's Equation

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Introduction

THE transonic small disturbance equation $(K-(\gamma+1)\varphi_x)\varphi_{xx}+\varphi_{yy}(x,y)=0$, in the usual notation, can be transformed into $-(\gamma+1)\phi_x\phi_{xx}+\phi_{yy}(x,y)=0$ using $\varphi=Kx/(\gamma+1)+\phi$. This latter equation is considered at great length in Guderley's classic monograph,[1] where various similar, separable and hodograph solutions are presented. In this Note, we re-examine Guderley's solution $\phi=x^3f(y)$ for the parallel sonic jet, and consider the broader class of separable solutions $\phi=g(x)f(y)$. Despite the apparent simplicity, solutions taking this form have not, to the author's knowledge, been treated previously. Guderley assumes $\phi=x^nf(y)$ and determines the exponent n so that separability is insured. The broader class of jetlike solutions considered here describes flows with more general expansion rates and should be of special engineering interest. In addition, the solutions provide useful test cases for numerical methods.

Analysis

The foregoing Ansatz leads to two ordinary nonlinear differential equations; namely,

$$f''-(\gamma+1)\lambda f^2=0 \tag{1}$$

$$g'g''-\lambda g=0 \tag{2}$$

where λ is a separation constant. Several solutions are possible. For example, two functions satisfying Eq. (1) are defined from:

$$f_1(y)=6/[\lambda(\gamma+1)]y^{-2} \tag{3}$$

and

$$\pm\int_{f^*}^{f_2}\frac{d\xi}{\sqrt{2\alpha+2/3\lambda(\gamma+1)\xi^3}}=y-y^* \tag{4}$$

where, in Eq. (4), f^* is the value of $f_2(y)$ at $y=y^*$ and α is a constant. Equation (2), on the other hand, is solved by $g_1(x)$ and $g_2(x)$, as determined from:

$$g_1(x)=(\lambda/18)x^3 \tag{5}$$

and

$$\int_{g^*}^{g_2}\frac{d\xi}{(3\beta+3/2\lambda\xi^2)^{1/3}}=x-x^* \tag{6}$$

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where g^* is $g_2(x^*)$ and β is a constant. Guderley's solution corresponds to $\phi = g_1(x)f_2(y)$ with $\lambda = 18$, and describes both the supersonic flow that results from the expansion of a parallel sonic jet and the case of a subsonic jet which gradually changes into a sonic one. The function $f_2(y)$ discussed and graphed in Ref. 1, will not be reconsidered here.

Other solutions are possible; for example, $g_2 f_2, g_1 f_1$, and $g_2 f_1$. The latter two are singular like y^{-2} on the axis and are probably of little physical interest. However, the solution $\phi = g_2(x)f_2(y)$ is meaningful and produces jets with alternative rates of expansion. Differentiation of Eq. (6) leads to $dg/dx = (3\beta + 3\lambda g^2/2)^{1/3}$. Let us introduce the nondimensionalizations $g(x) = A\tilde{g}(\tilde{x})$ and $x = C\tilde{x}$, where $C = 2^{1/2} |\beta|^{1/6} |\lambda|^{-1/2}/3^{1/3}$, and where $A = (2\beta/\lambda)^{1/2}$ if $\lambda\beta > 0$ and $A = (-2\beta/\lambda)^{1/2}$ if $\lambda\beta < 0$. Then, we are led to the following four allowable forms for $d\tilde{g}/d\tilde{x}$:

$$d\tilde{g}/d\tilde{x} = (1+\tilde{g}^2)^{1/3} > 0 \text{ if } \beta > 0, \quad \lambda > 0 \tag{7a}$$

$$= (1-\tilde{g}^2)^{1/3} \text{ if } \beta > 0, \quad \lambda < 0 \tag{7b}$$

$$= -(1-\tilde{g}^2)^{1/3} \text{ if } \beta < 0, \quad \lambda > 0 \tag{7c}$$

$$= -(1+\tilde{g}^2)^{1/3} < 0 \text{ if } \beta < 0, \quad \lambda < 0 \tag{7d}$$

Physically these solutions are easily interpreted. The horizontal velocity $\varphi_x = K/(\gamma+1) + g_2'(x)f_2(y)$ varies in the streamwise direction in a manner proportional to $d\tilde{g}/d\tilde{x}$. In the first case, Eq. (7a) gives a solution with ever-increasing $\tilde{g}(\tilde{x})$ in the positive x direction. Differentiation shows that $d\tilde{g}'(\tilde{x})/d\tilde{x} = 2\tilde{g}/(3(1+\tilde{g}^2)^{1/3})$. Thus, the factor $d\tilde{g}/d\tilde{x}$ in φ_x increases if $\tilde{g}$ is initially positive, so that the flow corresponds to an "accelerating" jet. If $\tilde{g}$ is initially negative, the rate of increase of $\tilde{g}'(\tilde{x})$ with respect to increases in $\tilde{x}$ is negative; the jet "decelerates," but since $\tilde{g}'(\tilde{x}) > 0$, it will accelerate once $\tilde{g}$ becomes positive. Similar considerations apply, obviously, to Eq. (7d).

Next consider Eq. (7b) for which $d\tilde{g}/d\tilde{x} = (1-\tilde{g}^2)^{1/3}$ and $d\tilde{g}'(\tilde{x})/d\tilde{x} = -2\tilde{g}/(3(1-\tilde{g}^2)^{1/3})$. The flow, therefore, accelerates or decelerates accordingly as $\tilde{g}'' > 0$ or $\tilde{g}'' < 0$. If $\tilde{g}$ is initially greater than 1, $\tilde{g}$ will decrease in the positive $\tilde{x}$ direction until $\tilde{g} \to 1+$. If $\tilde{g}$ is less than -1, $\tilde{g}$ will decrease and fall to $-\infty$. On the other hand, if $|\tilde{g}| < 1$ initially, $\tilde{g}$ will increase until $\tilde{g} \to 1-$, at which point $\tilde{g}$ will level off. As before, similar considerations apply to Eq. (7c). The axisymmetric form of Guderley's equation is also separable in the sense of this Note. The analogous $f_2(y)$ function is discussed in his monograph; the $g_2(x)$ function in that case is exactly the same as that treated in the forgoing analysis.

Closing Remarks

The preceding flows are obtained from a broad class of separable solutions and contain, as a special subset, Guderley's solution for the parallel sonic jet. They extend Guderley's results by allowing more general streamwise behavior, while at the same time retaining the same modal structure as given by his $f_2(y)$ crossjet function, and should be of considerable engineering interest. The solutions generated here should also provide useful check cases for numerical transonic algorithms.

References

[1]Guderley, K. G., *The Theory of Transonic Flow*, Addison-Wesley, Reading, Mass., 1962.

On Isotropic Two-Dimensional Turbulence

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In 1935, Taylor,[1] from the basic equations of continuity and energy dissipation in three-dimensional space, derived time correlations between space rates of change of velocities as well as an expression for the time mean dissipation of trubulent energy. It is the purpose of this paper to derive parallel relationships for two-dimensional isotropic turbulence. Definite differences occur that lead to simple interpretation. Furthermore, the von Kármán equation for correlation of velocity components in various directions shows that the Taylor length scale, $L = \int_0^\infty R_y dy$, is *always* zero for two-dimensional turbulence, a result which may throw doubt on the real significance of this definition.

Energy Dissipation

In x and y coordinates, with corresponding u and v velocity components, the equation for time-averaged energy dissipation per unit volume is:

$$W = \mu\left[2\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 2\overline{\left(\frac{\partial v}{\partial y}\right)^2} + \overline{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2}\right] \tag{1}$$

By isotropy,

$$\overline{\left(\frac{\partial u}{\partial x}\right)^2} = \overline{\left(\frac{\partial v}{\partial y}\right)^2} \text{ and } \overline{\left(\frac{\partial u}{\partial y}\right)^2} = \overline{\left(\frac{\partial v}{\partial x}\right)^2}$$

with which, upon expansion,

$$\frac{W}{\mu} = 4\overline{\left(\frac{\partial u}{\partial x}\right)^2} + 2\overline{\left(\frac{\partial u}{\partial y}\right)^2} + 2\overline{\left(\frac{\partial u}{\partial y} \cdot \frac{\partial v}{\partial x}\right)} \tag{2}$$

Hence, further reduction requires relations between mean squares and mean products.

Following Taylor precisely, there are only so many combinations of $\partial u/\partial x$, $\partial v/\partial x$, $\partial u/\partial y$, and $\partial v/\partial y$; namely,

$$\frac{n!}{2!(n-2)!} = \frac{4!}{2!\cdot 2!} = 6$$

With the four mean squares, the total number of mean products to reckon with is ten. Thus,

$$\overline{\left(\frac{\partial u}{\partial x}\right)^2}, \overline{\left(\frac{\partial v}{\partial x}\right)^2}, \overline{\left(\frac{\partial u}{\partial y}\right)^2}, \overline{\left(\frac{\partial v}{\partial y}\right)^2}, \overline{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x}}, \overline{\frac{\partial u}{\partial x}\cdot\frac{\partial u}{\partial y}},$$

$$\overline{\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y}}, \overline{\frac{\partial v}{\partial x}\cdot\frac{\partial u}{\partial y}}, \overline{\frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial y}} \text{ and } \overline{\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y}}$$

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